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EFFECT OF THE GROUND ON AN AIRPLANE FLYING CLOSE TO IT

By E. Tönnies

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INTRODUCTION

While taking off and landing or, in general, while flying near the ground, the flight characteristics of an airplane are affected by the nearness of the ground, which will here be taken to mean within a few meters of the wing.

It is a well-known fact that a low-wing airplane takes off quicker than an equivalent high-wing airplane, due to the greater "ground effect" on the low wing. Likewise the following noteworthy observation, which has been made in recent years in the taking off of very heavily loaded airplanes on long flights, e.g., in ocean crossings, is attributable to the ground effect. The limit between the maximum load with which an airplane can take off and the load with which it can no longer take off does not appear to be very sharply defined, but depends on another possibility expressed by the fact that the airplane can leave the ground after taxying a long distance and is then unable to climb higher than about half the wing span for a long distance, even as much as ten miles according to an American report. (Reference 1.) Such cases have repeatedly occurred and are attributable to the ground effect in so far as a slight increase in the total load is offset by the improved lift-drag ratio near the ground. (Reference 2.)

A similar phenomenon is also observed in landing. An airplane can fly a long distance near the ground even after its speed has diminished to the point that would prevent it from ascending. It is reported by pilots, however, that in this condition an airplane often pancakes without apparent cause. Probably the air flow suddenly

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*"Der Boden-Effekt beim Fluge in Erdnähe." Zeitschrift für Flugtechnik und Motorluftschiffahrt, March 29, 1932, pp. 157-164.

separates from the wing near the ground.

In America and England the ground effect has received much attention for a long time, and whole series of model and flight tests have been made, which have, however, been chiefly devoted to changes in the induced drag. The experiments here described show that the increase in lift may be of a high enough order of magnitude to be taken into account also.

CONSIDERATION OF TAKE-OFF CONDITIONS FROM STATISTICAL DATA

It was first attempted to determine statistically the difference in the take-off runs of high-wing and low-wing monoplanes and of biplanes from the data obtained by the D.V.L. (Deutsche Versuchsanstalt für Luftfahrt) in their own take-off tests with various airplane types. The airplanes were placed at the disposal of the D.V.L. with the consent of the manufacturers. At the outset, however, attention is called to the fact that the figures given here are only intended to show a general tendency and cannot be used for accurate calculations. Hence the following aspects of the starting conditions will be briefly discussed, in order to show how many factors, some of which can only be estimated, affect the calculation of the take-off distance and time, and how difficult and hazardous it is to compare the take-off performances of different airplane types flown by different pilots under different conditions.

In the course of time a whole series of graphic and analytic methods for the calculation of take-off data has been developed. The views here expressed are based on the formulas developed by Blenk. (Reference 3.)

The derivation of the take-off formulas is based on the fundamental principle of dynamics that the force equals the mass times the acceleration, so that

$$\frac{G}{g}\frac{dv}{dt} = S - W - R \qquad (1)$$

After making several simplifying assumptions and integrating, we obtain the following equations for the take-off time and distance:

$$T = \sqrt{\frac{1}{g\,\frac{S_0 - \mu G}{G}\left(\frac{\epsilon}{F} + c_{w1} - \mu\, c_{a1}\right)\frac{\gamma}{2}\,\frac{F}{G}}} \times$$

$$\underline{\text{arc}}\ \underline{\tan}\left(v_2\sqrt{\frac{\left(\frac{\epsilon}{F} + c_{w1} - \mu\, c_{a1}\right)\frac{\gamma}{2}\,\frac{F}{G}}{g\,\frac{S_0 - \mu G}{G}}}\right) \qquad (2)$$

$$s = \frac{2\ G}{\left(\frac{\epsilon}{F} + c_{w1} - \mu\, c_{a1}\right)\gamma\ F}$$

$$\ln \frac{1}{\sqrt{1 - \left(\frac{G}{S_0 - \mu G}\,\frac{\frac{\epsilon}{F} + c_{w1} - \mu\, c_{a1}}{c_{a2}}\right)}} \qquad (3)$$

G being the flying weight, S_0 the propeller thrust, W the air resistance, R the frictional resistance, F the wing area, ϵ the reduction factor of the propeller thrust, μ the viscosity coefficient, c_{a1} and c_{w1} lift and drag coefficients in taxying, c_{a2} corresponding to $\left(\frac{c_w}{c_a^{1.5}}\right)_{min}$, γ the air density and v_2 the best climbing and take-off speed.

The take-off is obviously affected by a whole series of factors including several which cannot be accurately determined for each case, as, for example, the lift and drag coefficients, the propeller thrust S_0 and the viscosity coefficient μ. The lack of the exact value of μ is the cause of the largest and most frequent errors. There is still another factor which does not appear in the calculation, namely, the personal equation of the pilot.

In order to eliminate as much as possible, in the comparison of the different values, any contingencies during the tests, like gusts or peculiarities in piloting, the take-off distance s was calculated, for all the airplanes to be compared, according to an approximation formula also developed by Blenk:

$$s = \frac{G^2}{\gamma\ F\ c_{a_2}\ (S_0 - \mu G)} \tag{4}$$

or, by introducing v_2 (take-off) $= \sqrt{\frac{G}{F} \frac{2g}{\gamma\ c_{a_2}}}$,

$$s_r = \frac{v_2^2\ G}{2g\ (S_0 - \mu G)} \tag{4a}$$

whereby it must be assumed that the taxying is continued until the best climbing speed (v_2) is reached, so that level flight or "floating" after the take-off is entirely eliminated. Then the measured take-off distance s_v (including taxying and floating), is plotted against the calculated take-off distance s_r. This yields a 45° straight line through the origin, if the values used in the calculation correspond to the real values. Hence, if it is assumed that the value of S_0 according to the formula

$$S_0 = 4\ N \sqrt[3]{\frac{F}{N}} \quad \text{(Reference 3)} \tag{5}$$

corresponds approximately to the facts and further that the value adopted for μ is the correct one, v_2 being taken from the tests, then any deviation from this straight line must be due to the above-mentioned contingencies. In the calculation μ was assumed to be 0.15 which, according to Figure 1, closely approximates the actual value, while the value $\mu = 0.1$, generally considered the practical mean, is probably a little too low, at least for an ordinary airplane without a runway. The ever-present deviation may be due to the fact that, in the first place, the most favorable manner of taking off is assumed in the calculation and, secondly, that the approximation formula represents only the simplified first term of a series development of the accurate formula (3).

In the further consideration we then used only the values which deviated but slightly from the continuous straight line. Notwithstanding the elimination of the contingent values, it is always difficult to compare different airplane types, since the constructive factors which affect the take-off, such as wing loading, power loading and the ratio of the propeller thrust to the weight, are different for all of them. It was attempted to represent the effect of all these values by plotting (fig. 2) the necessary taxying distance, in meters per unit power load-

ing, against the expression $S_0 - \mu G$, that is, the excess power used for the acceleration, corresponding to the approximation formula.

If the above-mentioned difference in the aerodynamic characteristics of high-wing and low-wing monoplanes in taking off actually existed, it would be shown by the plotted values not being all on one curve. If we should start with the assumption that the ground effect either increases the lift or decreases the induced drag, the ground pressure and the air resistance would decrease more rapidly and the acceleration-producing force and the take-off run would both be smaller, so long as the wing is in the region of the ground effect. During level flight near the ground (or "floating"), a low-wing monoplane becomes, as it were, a high-wing monoplane and the same conditions hold good for both. If we should disregard the fact that, while floating, the ground friction is eliminated and the requisite speed for climbing is reached somewhat quicker, the sum of the taxying and floating distances would be the same for both airplane types, as shown in Figure 1. It is obvious from Figure 2, however, that the taxying distance per unit power loading is actually shorter for the low-wing monoplane with the same available excess power, which is ascribable to the ground effect.

DESCRIPTION OF THE EXPERIMENTS AND THEIR RESULTS

Track with Test Carriage

No quantitative conclusions can yet be drawn from the above-mentioned experiments regarding the effect of the proximity of the ground on the polar of a wing. It cannot even be determined whether the above-mentioned facts are due to increased lift or decreased drag or a combination of both, which is more probable. The following is a report of model and flight tests, the results of which will subsequently be compared with the theory.

Model tests.- Unfortunately, Hannover has no wind tunnel of its own, so that the tests had to be made with a carriage running on rails. The carriage supported a wing model at a sufficient distance in front and was operated by a falling weight. (Figure 3.)

Since, for the sake of economy, all the apparatus had to be made by hand, the Göttingen profile No. 365 was chosen for the wing model. This was flat on the lower side and was therefore easier to make than a perhaps aerodynamically more favorable airfoil with a concave lower surface. Its dimensions were 20 by 100 cm (about 8 by 40 inches). The wing was supported by a system of rods, as on a balance, and could be set at different distances from the ground plane. The horizontal and vertical motions of the wing were automatically recorded by a stylus on carbon paper wound on a drum operated electrically by clockwork, whereby calibrated springs were stretched, so that the magnitude of the deflections served also as a criterion for the forces acting on the wing. (Fig. 4.)

Tests were made for each angle of attack at different distances from the ground plane. Unfortunately the available space was only 22 m (about 72 feet) long, so that, with the most favorable division into starting run test distance and stopping run, a speed of only 6.5 m (21.3 ft.) per second could be attained. The 5 m (16.4 ft.) was therefore traversed in 0.77 second. In order to measure this short period as accurately as possible, a device was constructed which operated as follows. A simple bell magnet recorded the vibrations imparted to it by a 50-period alternating current on a carbon paper attached to a clockwork drum. Under this vibration curve with 100 complete vibrations per second, another bell magnet recorded deviations due to current impulses produced by the test carriage passing over sliding contacts at definite intervals. (Fig. 5.) The speed over the whole test distance could be very accurately determined from meter to meter by counting the vibrations. The speed was determined for every test.

The force acting on the wing and the corresponding speed were determined from the two diagrams 4 and 5, and the value of c_a was calculated according to the well-known formula for the lift $A = c_a F q$. Allowance had to be made, however, for the fact that the carriage did not move at a uniform speed over the test distance, but was slightly retarded by friction and the resistance of the air, as could also be determined from diagram 5. The lift A consisted of the two factors, the spring elongation K and the inertia forces M produced by the retardation. These inertia forces could be readily calculated for any position of the wing, since the masses and their lever arms were known. Table I shows the process of calculation, only two values being taken for lack of space.

TABLE I

Angle of attack α^o	Height of wing h in mm	h/b	Speed V m/s	Force of spring A g	Inertia force K g	A-K g	c_a
4^o	58	0.116	6.4	540	134	406	0.755
4^o	243	0.886	6.45	460	125	335	0.625

The final results are plotted in Figures 6-8. In Figure 6, c_a is plotted against the ratio h/b, that is, twice the distance of the wing from the ground to the span. It is obvious how, with increasing nearness to the ground, the lift increases beyond its normal value and indeed most at small angles of attack, while there is a slight decrease at large angles of attack corresponding to $c_{a\ max}$. Perhaps this is connected with the above-mentioned pancaking while flying level near the ground, because the pilot levels off shortly before setting the airplane down and thus comes within the angle-of-attack range where there is no further lift increase, such as there was before he leveled off. In Figure 7 the c_a values are plotted against the angle of attack for various ratios of h/b as parameter, the percentile lift increase over its normal value at unaffected altitudes being also shown. Figure 8 compares the Göttingen wind tunnel results with those obtained with the test carriage for the same wing profile at an unaffected distance from the ground and shows that very good results can be obtained with a test carriage by exercising sufficient care. The slight discrepancy between the two test results are ascribable to the fact that the hand-made wing model did not have exactly the same shape as the Gottingen model, though made from the same measurements.

Since it has repeatedly been established by both model and flight tests (reference 4) that the formulas proposed by Wieselsberger (reference 5) for calculating the induced drag of a wing in proximity to the ground, as derived from Prandtl's multiplane theory, agree very well with the experimental results, only tests for determining the lift variation were here made. The drag values used farther along were calculated by Wieselsberger's method.

Accuracy of the apparatus.- The speed could be deter-

mined with any desired degree of accuracy, since the time consumed by the test carriage in traversing the test distance could be readily measured to 0.01 second. As shown in Figure 4, the lift curve, scratched in the carbon coating with a pointed stylus, is very fine, making it possible to measure the distance from the zero line to within 1/4 mm (0.01 in.). This is a criterion for the lift, where 1/4 mm would be equivalent to about 10 g (0.022 lb.) corresponding to an error limit of ±0.035 g (0.00008 lb.). On a rather large scale, the angle of attack could be accurately determined to within 1/4 degree.

Flight Tests and Their Results

Flight tests have been conducted in America for the numerical determination of the ground effect. (Reference 1.) In these tests, however, only the effect on the induced drag was considered. It was found that, with a given propeller thrust, a greater speed could be attained in flight near the ground than at a higher altitude. An increase of 1.3 per cent in the speed was observed while flying with the lower wing of a biplane only 5 to 7 feet from the ground. Unfortunately, no data are given regarding the angle of attack, so that it is impossible to tell from the experimental polar whether there was any change in the lift.

Flight tests were made in Hannover with the low-wing monoplane of the Klemm 26-2a type, for the purpose of determining whether the lift variation observed in model tests also occurred with full-scale airplanes. We again have

$$c_a = \frac{G}{F \frac{\gamma}{2\,g}} \frac{1}{v^2} = K \frac{1}{v^2} \qquad (6)$$

where G denotes the flying weight, F the wing area, γ the air density and g the acceleration due to gravity combined in a constant K, v the horizontal speed and c_a the lift coefficient. For flight near the ground the formula would be

$$c_a' = K \frac{1}{{v_1}^2} \qquad (6a)$$

Here $c_a' = c_a + \Delta c_a$, and v' would represent a speed

correspondingly smaller than v. This would indicate that, near the ground, one could fly either at the same angle of attack and a lower speed or at the same speed and a lower angle of attack, as compared with flight at an altitude free from ground effect. In the experiments, therefore, the speed and angle of attack had to be measured, as likewise the height of the wing above the ground and the velocity of the wind. These quantities were determined photographically from the ground by means of a new Zenith camera kindly placed at our disposal by the Ascania Works at Berlin-Friedenau. This camera was specially adapted for photogrammetric flight tests.

The experimental arrangement was as follows: At a distance of about 160 m (525 ft.) from the camera, three surveyor's rods were stuck into the ground 50 m (164 ft.) apart, so as to form a straight line in the direction of the wind. The line connecting the camera with the middle rod was perpendicular to this straight line. The task of the test-plane pilot was to fly as closely as possible to these rods, the recognition of which was facilitated by directional signs on the ground, while the photographer followed him with the finder, and exposures were automatically made every second on the same plate. (Fig. 9.) After several practice flights, the pilot succeeded in making a series of flights in the desired direction with the wing only one meter (about 40 in.) and the wheels only 10 to 20 cm (4 to 8 in.) above the ground. Since these test flights were very dangerous, only so many were made as were necessary to furnish the desired proof of a lift increase, i.e., since the flying weight remained the same, test flights were made only for a relatively small angle-of-attack range, namely, from -1° to $+1^\circ$. Flights were thus photographed at altitudes of 2, 4, 7, 10, 15 and 20 meters (6.5 to 65 feet), the analysis showing that even at 7 m (23 ft.) there was no measurable ground effect. In each test the three surveyor's rods were included in the photographs, in order to determine the exact height of the wing above the ground and also the horizontality of the flight. Then, for comparison, a flight at 25 to 30 m (82 to 98 ft.) altitude was photographed on the same plate. The results plotted in Figure 11 were obtained on a clear winter's day with a light snowfall on the ground and absolutely no wind, which is very favorable for the interpretation, because all errors due to wind fluctuations are eliminated. Only the two altitudes of 2 and 4 m (6.5 and 13 ft.) were measured in a wind and calculated for no wind.

For evaluation, the photographs were projected on millimeter paper and the distinguishing points of the airplane, such as the propeller hub, trailing edge of the rudder, bottom of wheel and lower edge of tail were marked on the paper. (Fig. 10.) With the size of the airplane, exposure interval, focal length of camera and enlargement ratio of projection known, it was possible to determine the speed to within a few tenths of a meter per second and the angle of attack to within 1/6 of a degree. It was also possible to determine the height of the wing above the ground with the aid of the known height of the surveyor's rods. Of the 12 to 15 pictures covering the entire length of the plate, only four or five were used for the evaluation, namely, the ones near the middle rod, in order to avoid the distortion of the distances and angles due to the perspective.

Figure 11 shows the result of the tests, in which c_a, calculated according to formula (6), is plotted against the angle of attack for a height of the wing of 25 m (82 ft.) above the ground, as free from ground effect, and of one meter (about 40 in.), that is, $h/b = 0.155$, as the shortest practicable distance from the ground for the airplane to fly. The lift increase is plainly shown. We obtain, e.g., at 1° angle of attack and $h/b = 0.155$, an increase of 10.3 per cent as compared with the normal c_a value, which, though considerable, is not so large as that indicated by the model tests, which is about 35 per cent for the same angle of attack and the same value of h/b. This discrepancy may be due to the fact that the ground effect is disturbed by the fuselage and propeller slipstream and cannot therefore attain so great a value as for the wing model.

GROUND EFFECT ON TAKE-OFF AND LANDING

It has already been established, on the basis of model and flight tests that the proximity of the ground affects the wing polar in the sense that the lift is increased, as compared with the normal lift, and indeed the most at small angles of attack, while the induced drag is reduced at large angles of attack (as calculated by Wieselsberger's method). We will now consider the effect of this phenomenon on the take-off and landing characteristics of an airplane.

Figure 12 shows the polar of the wing used for the

tests described in the preceding section. The continuous line is the polar for the unaffected altitude, which will be called the normal polar. The short-dash curve represents the polar calculated according to Wieselsberger for $h/b = 0.1$, in which the lift was assumed to remain unchanged, as shown by the fact that the angles are at the same height for both curves, while in the third curve the changes in the c_a values are also considered.

We will now illustrate by an example the ground effect on the take-off of an airplane. We will take a low-wing monoplane with $h/b = 0.1$ and the following dimensions: $G = 500$ kg (1102 lb.) flying weight, $F = 20$ m² (215.3 sq.ft.) wing loading, $N = 70$ hp, $S_o = 184$ kg (406 lb.) propeller thrust on stand, $S = 90$ kg (198 lb.) propeller thrust at $v = 35$ m/s (115 ft./sec.), $\epsilon = 1.2$ reduction factor of propeller thrust, $\mu = 0.1$ coefficient of friction. It is also assumed that the taxying is done at the angle of attack corresponding to the best climbing flight, so that $c_{a2} = c_{a1}$ and $c_{w2} = c_{w1}$ and that the transition from taxying to climbing will occur without floating. The index n indicates the normal polar and b the polar affected by the nearness of the ground. The values in Table II were calculated according to formulas (2) and (3).

TABLE II

	Normal	Affected
c_{a2} corresponding to $\left(\frac{c_w}{c_a^{1.5}}\right)_{min}$	0.914	0.975
c_{w2}	0.095	0.060
Best take-off and climbing speed v_2	21.6 m/s (70.9 ft./sec.)	20.8 m/s (68.2 ft./sec.)
Take-off time t	9.0 s	8.1 s
Take-off distance s	95.0 m (312.0 ft.)	78.0 m (256.0 ft.)

If it be assumed, for example, that an airplane can take off both as a low-wing and as a high-wing monoplane, the latter, due to the ground effect, would require, according to the table, an 18.5 per cent longer take-off run

than the former. A graphic representation of the same example shows the relative effects of the increased lift and reduced drag due to the ground effect. (Reference 6.) In Figure 13 all the forces acting on the airplane during the take-off are plotted against the speed. The continuous curves correspond to the normal polar and the dash curves to the affected polar. This figure shows two facts: first that the drag curve is lower with the use of the polar affected by the nearness of the ground, just as the curve of the frictional forces R, in that, due to the higher c_a value, the ground pressure and friction drop faster toward zero. The sum of both curves (W + R) lies, of course, somewhat lower, whereby the force P, available for the acceleration, is increased. Secondly, the requisite speed for climbing is reduced by the better climbing ratio and is more quickly attained. Both factors cooperate to reduce the take-off run for the low-wing monoplane. The time interval Δt is here assumed to be one second. It is obvious that, in using the affected value, the number of triangles formed by the zigzag line is smaller and consequently the take-off time is less. We obtain the values $t_n = 9$ s and $t_b = 8$ s, which agree with the above-calculated values.

The following is an addition to the many observations already made regarding the most favorable take-off. (Reference 7.) In general, two principal assumptions are made: First, that, in taking off, taxying is continued until the speed v_2, corresponding to the best climbing ratio, is attained; secondly, that the whole distance is traversed at a constant angle of attack. The first assumption is justified by the fact that it can be established, both theoretically and practically, that the take-off will be the shortest when the floating distance is kept as small as possible. (Reference 8.) The second assumption, as Blenk has shown (reference 3), is derived from the take-off formula (3), from which a minimum is obtained when the factor $(c_w - c_a)\,\mu$ is a minimum. This is the case when $\frac{d\,c_w}{d\,c_a} = \mu$. In order to find the corresponding angle of attack at which the taxying must be done, it is only necessary to draw a tangent to the polar with the inclination μ. The contact point gives the c_a and c_w values for the shortest take-off, but does not need to agree with the values for the best climbing ratio.

In Figure 14 this method is applied to the foregoing

example for both the normal and the affected polar. It is seen that the tangent to the polar affected by the ground yields lift and drag values very different from the most favorable ones. The values obtained from the figure are: for the normal polar, a minimum of $s_n = 93.5$ m (306.8 ft.) and, for the affected polar, $s_b = 76.5$ m (251 ft.). Both values are smaller than the above-calculated ones, for which it was assumed that the taxying was at an angle of attack corresponding to $(c_w/c_a^{1.5})_{min}$.

In the same figure the normal polar is plotted for another aspect ratio $F/b^2 = 1/10$. The induced drag is known to be smaller in proportion as the ratio F/b^2 is smaller, the polar moving to the left and becoming steeper. For this case another tangent with the inclination $\mu =$ 0.1 was drawn to the polar. It is seen that the contact point is higher, thus improving the ratio of the lift coefficient to the drag coefficient. The values thus obtained yield a minimum take-off distance of $s = 90$ m (295 ft.), which is less than for the first polar.

The result of the foregoing considerations is therefore that the take-off distance is the shortest, when the wing is closest to the ground and the ratio F/b^2 is the smallest, provided that the whole take-off distance is traversed at the best angle of attack and that floating is eliminated.

ON THE THEORY OF FLIGHT NEAR THE GROUND

For completeness and comparison we will include the results of a theoretical investigation of flight near the ground by J. Bonder of Warsaw. (Reference 9.) Bonder works out very complex mathematical formulas by proceeding from the flow relations of two adjacent cylinders with the aid of conformal transformation to two opposite wing profiles separated by a plane of symmetry (the ground). (Compare also Wieselsberger's theory of the induced drag for this case.) Bonder thus arrives at a formula which renders it possible to calculate the forces acting on both wings, perpendicular to the direction of flow and therefore identical with the lift, for different angles of attack and various distances between the wings.

Since this formula is too troublesome for numerical calculation, Bonder suggests a more convenient approxima-

tion formula and shows by an example that its results differ by only a few per cent from the accurate formula. The approximation formula reads:

$$\frac{P_y}{r\ v_o^{\,2}} \approx \frac{\rho \Gamma}{r\ v_o} = K \left[\frac{1}{4} + \sum_{n=1}^{n=\infty} \frac{(-1)^{n+1}}{e^{(2n-1)\eta_o} - 1}\right] \tag{6}$$

The factor K is calculated from

$$K = 2\ \frac{2 \sin\delta\ \dfrac{\cos \eta_o - \sin\delta}{\sin \eta_o} - \dfrac{\Gamma \eta_o}{2\pi v_o r} - 4 \sin \eta_o \displaystyle\sum_{n=1}^{n=\infty} \frac{n \cos n\ \xi_o}{e^{n\ \eta_o}(e^{2n\ \eta_o} - 1)}}{1 + 8 \displaystyle\sum_{n=1}^{n=\infty} (-1)^{n+1} \frac{n}{e^{2n\ \eta_o} - 1}} \tag{7}$$

in which

$$\cos \xi_o = \cos \eta_o - \frac{\sin^2 \eta_o}{\cos \eta_o - \sin \delta} = \frac{1 - \cos \eta_o \sin \delta}{\cos \eta_o - \sin \delta}$$

Here δ denotes the angle between the vortex trail behind the wing and the direction of the velocity v_o in infinity; η_o is the mirrored circle of the cylinders. For $\eta_o = \infty$, the distance in which the cylinders are infinitely separated from one another, $\delta = \delta_o$. At this angle the circulation $\Gamma = 0$ and consequently the lift is also zero. The inclination of the profile and of the vortex trail to this zero position is expressed by the angle β

$$\beta = \delta - \delta_o .$$

Figure 15 shows the increase in the circulation on approaching the ground for various angles β. For the case when $\eta_o = \infty$, that is, when the wing is at an undisturbed distance from the ground, Γ is obtained from formula (6) for $\eta = \infty$ or $h = \infty$ and correspondingly, $K = 4 \sin \beta$ with the air density $\rho = 1/8$.

$$\Gamma_{h=\infty} = 8\ r\ v_o \sin \beta \tag{8}$$

Let the ratio of $\Gamma/r\ v_o$ for a finite distance h of the wing profile from the ground to the same expression for $h = \infty$ be λ. Since $\Gamma_\infty/r\ v_o = 8 \sin \beta$, this ratio is

$$\lambda = \frac{\Gamma}{r\ v_0}\ \frac{1}{8 \sin \beta} \tag{9}$$

In Figure 16, $\lambda = f(h)_{\beta=\mathrm{constant}}$ is plotted for various angles β = constant.

In order to make a general comparison of the results obtained from this theory with the experimental results, the following facts must be considered. In every case the wing chord must come within the limits $2r$ and $4r$, the former being for the cylinder and $4r$ for the flat plate. Since the most commonly used airfoils are relatively flat, r is about $1/4$ t. The angle $\beta = 0^\circ$ is the one at which the lift is zero. In the case of the airfoil used in the experiments, $\alpha = 0^\circ$ is therefore approximately identical with $\beta = 6^\circ$. Moreover, h is here the distance of the wing from the ground, not twice the distance as before. For $\alpha = 0^\circ$ and $h/r = 1$, corresponding to $h/b = 0.1$, a lift increase of 85 per cent is obtained from Figure 16, as compared with only 40 per cent obtained experimentally. This difference may be due to the fact that an infinite span was assumed in the theoretical consideration of the wing.

The important point of the theoretical results is the evidence, in agreement with the experimental results, of the lift increase of a wing on approaching a flat surface and of such an order of magnitude as not to be negligible. (Reference 10.)

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

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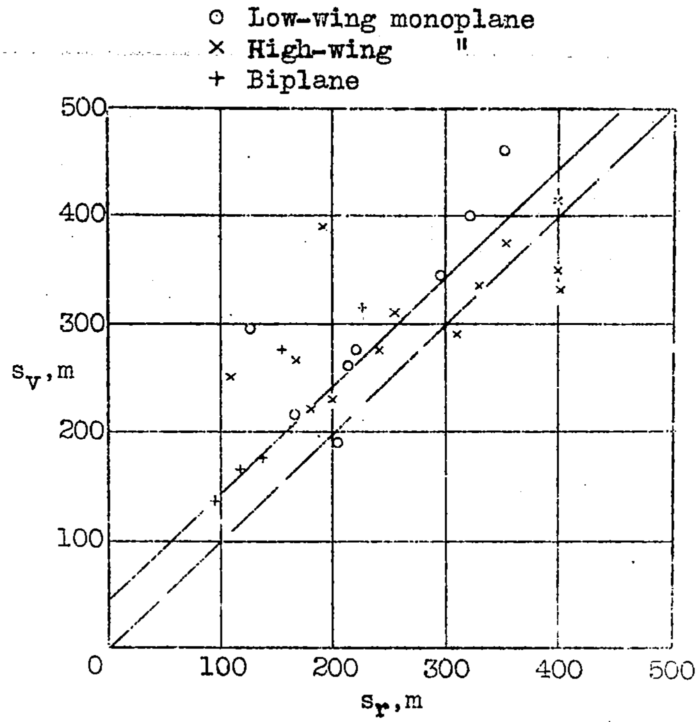


Fig.1 Comparison of measured take-off distances s_v of different airplane types with the calculated distances s_r (taxying + floating.)

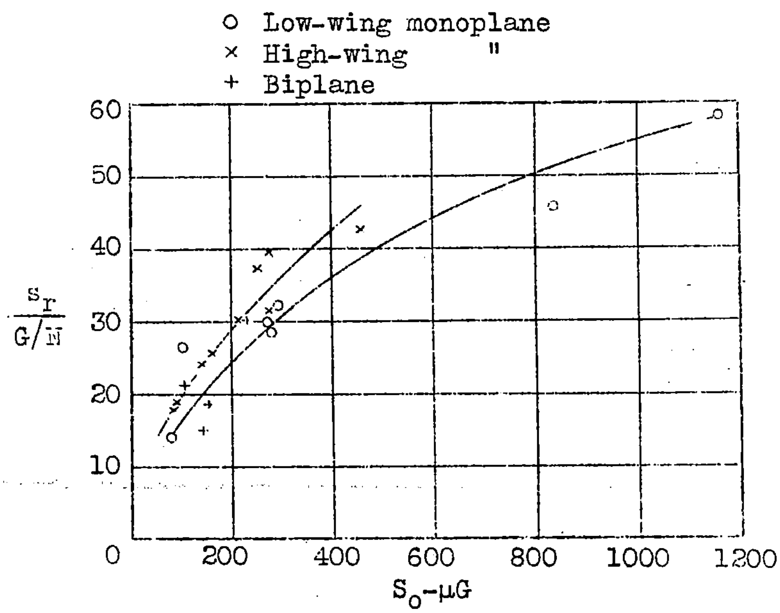


Fig.2 Necessary taxying distance in meters per unit power loading with a given excess power (in kg). (Experimental values).

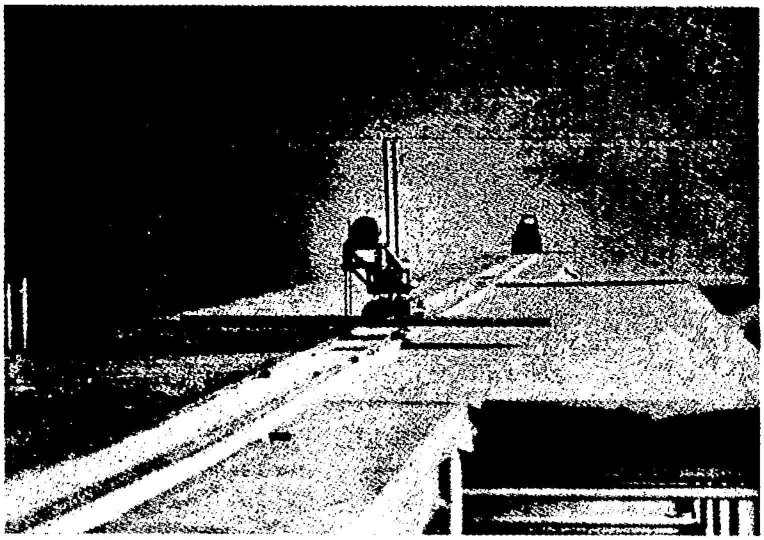


Fig.3 Track with test carriage.

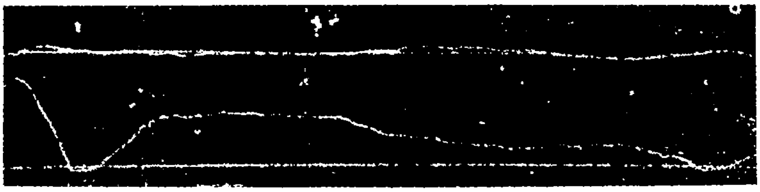


Fig.4 Lift diagram.

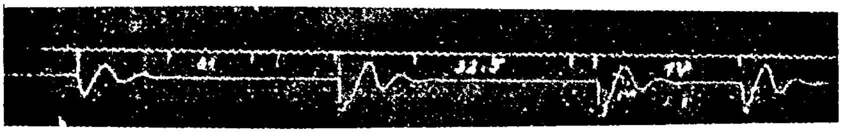


Fig.5 Time measurement.

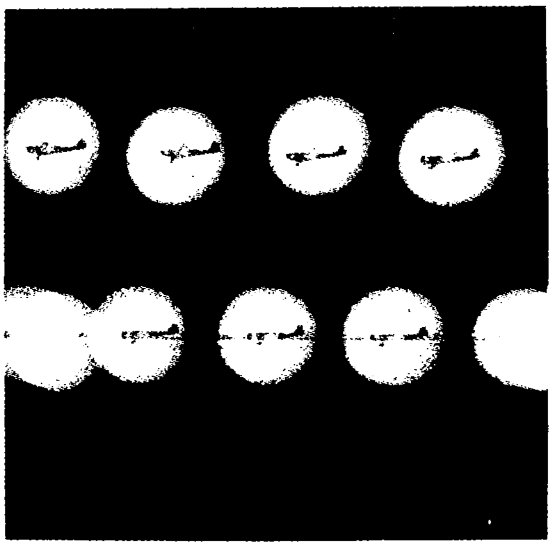


Fig.9 Section of picture series taken with Zenith camera. Lower flight was at 1 m (3.28 ft.) from ground. Time interval between pictures 1/2 second.

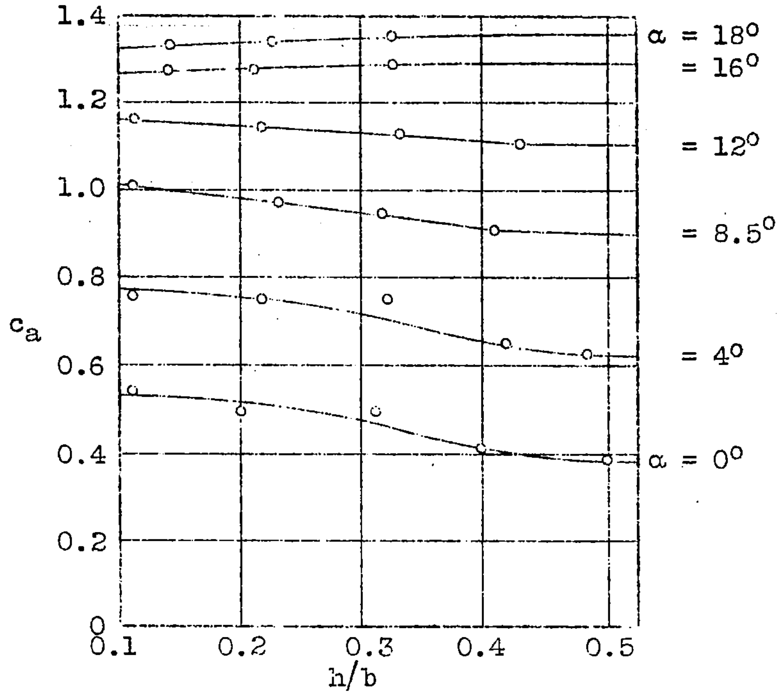
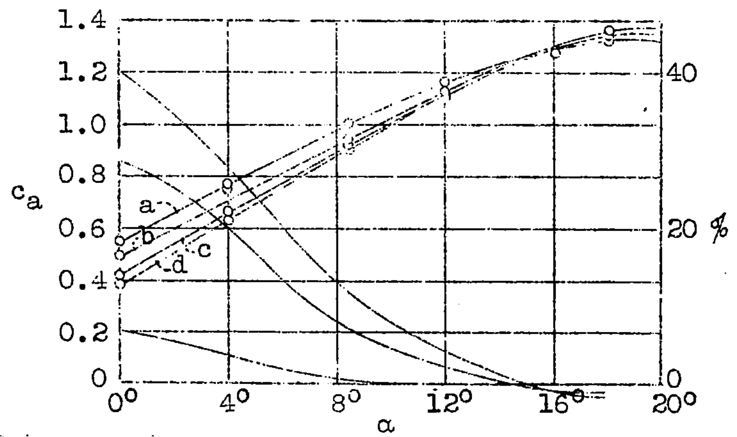


Fig.6 Variation in lift as wing approaches a flat surface.



a, h/b = 0.1 b, h/b = 0.3 c, h/b = 0.4
d, Unaffected

Fig.7 Lift coefficients plotted against angle of attack at various distances of wing from ground. Percentile lift increases in comparison with normal lift at unaffected distance of wing from ground.

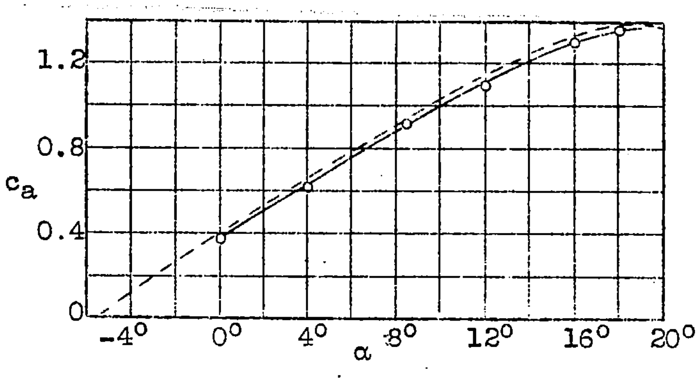


Fig.8 Comparison of test-carriage results with Göttingen wind-tunnel tests of the same airfoil.

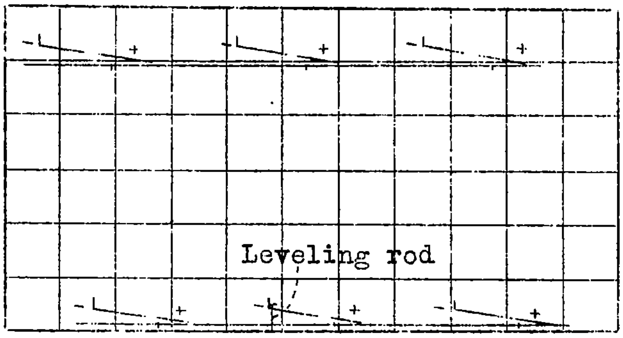


Fig.10 Evaluation of projected pictures.

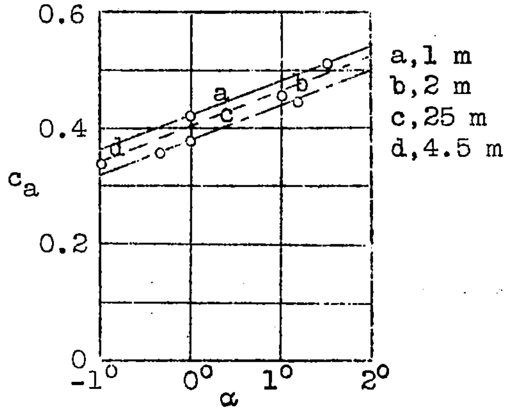


Fig.11 Lift coefficients at various distances from the ground. (Flight tests).

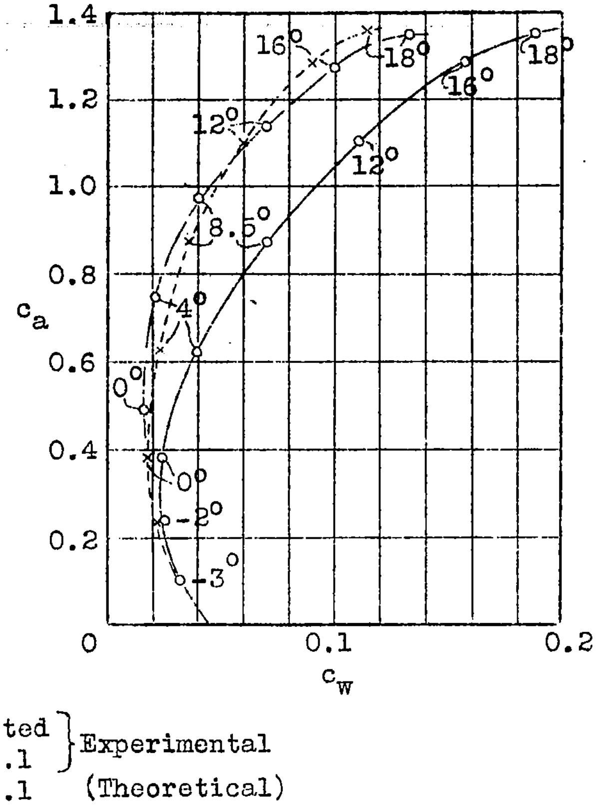


Fig.12 Wing-model polars. (Test carriage results).

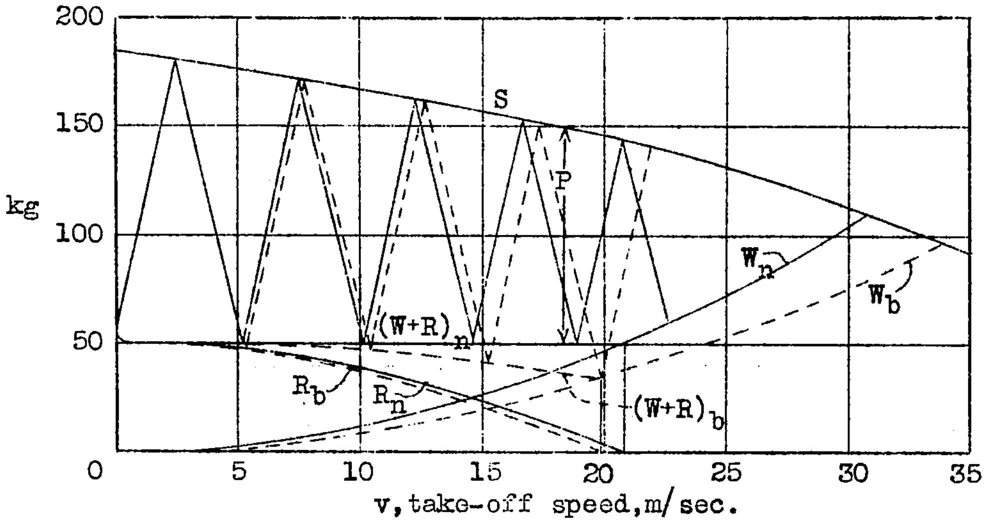


Fig.13 Graphic representation of take-off. Comparison of high-wing (n) and low-wing (b) monoplanes.

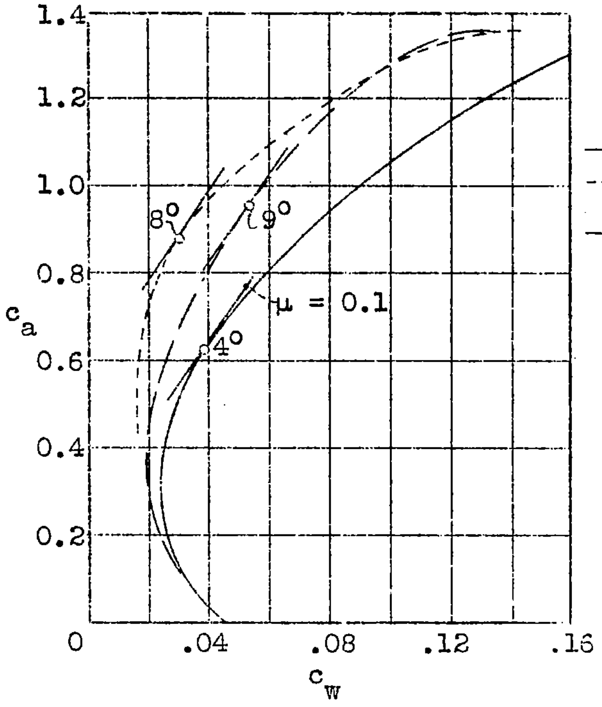


Fig.14 Determination of lift and drag coefficients for shortest take-off.

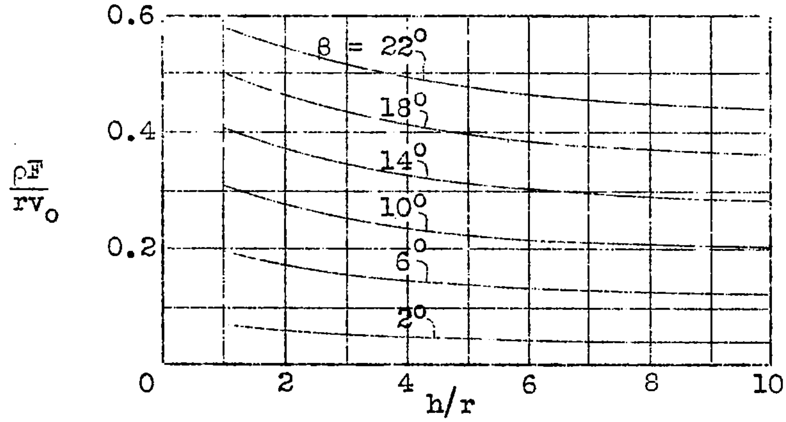


Fig.15 Theoretical circulation increase of wing on approaching a level surface.

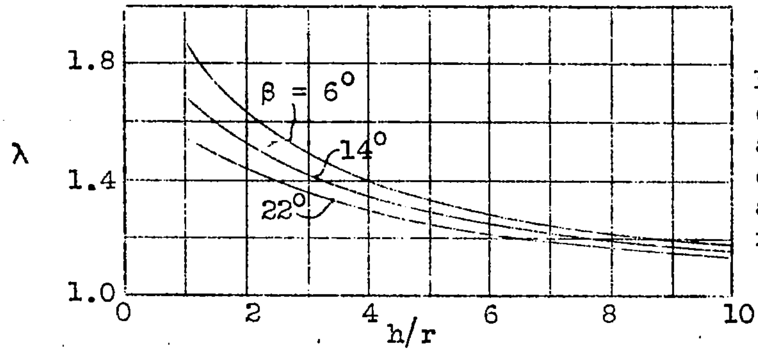


Fig.16 Ratio of circulation of wing at finite distance to circulation of wing at infinite distance from ground.